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**UNIVERSITÀ  
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## Scientific Program - Timetable

Sun day 22	Time	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
	9:15-9:45		Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
	10:15-10:45			von Mises prize lecture		
	11:15-11:45	Registration	Coffee Break	Coffee Break	Coffee Break	Coffee Break
	12:15-12:45		Plenary Lecture Thomas Böhlke	General Assembly	Plenary Lecture Ferdinando Auricchio	Contributed sessions (11 in parallel)
	13:15-13:45		Opening	Lunch	Lunch	
		Univ. Chorus Performance				Closing
	14:15-14:45	Prandtl Lecture Keith Moffatt	Plenary Lecture Enrique Zuazua	Contributed sessions (15 in parallel)	Plenary Lecture Daniel Kressner	
	15:15-15:45	Plenary Lecture Giovanni Galdi	Plenary Lecture Nikolaus Adams		Plenary Lecture Stanislaw Stupkiewicz	
Registration pre-opening	16:15-16:45	Coffee Break	Coffee Break Poster session	Coffee Break	Coffee Break Poster session	
	17:15-17:45	Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Contributed sessions (14 in parallel)	Contributed sessions (15 in parallel)	Contributed sessions (15 in parallel)	
	18:15-18:45					
	19:15-19:45	Opening reception at Castle of Charles V	Public lecture Francesco D'Andria			
	20:15-20:45					
	21:15-21:45			Conference dinner at Hotel Tiziano		

# GAMM 2015

Università del Salento

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## **S15: Applied stochastic**

The session welcomes contributions on the following topics: uncertainty quantification; risk analysis and assessment; Bayesian methods in engineering; decision analysis; probabilistic modelling; stochastic mechanics. Applications to large scale engineering problems involving the use of computer models, like fluid mechanics and structural mechanics, are encouraged.

# Efficient Bayesian analysis of rare events in numerical models

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Bayesian analysis is a consistent and effective framework for combining new information with existing models, in which prior probabilistic models are updated with new data and observations. For engineering models, Bayesian analysis can be interpreted as a stochastic inverse analysis. The Bayesian framework allows the engineer to combine uncertain and incomplete information with models from different sources. Moreover, it provides probabilistic information on the accuracy of the updated model.

When performing Bayesian analysis of engineering models, numerical methods such as Markov Chain Monte Carlo (MCMC) are necessary to evaluate the posterior distribution of the model parameters and to make predictions with the posterior model. Recently, the authors proposed a novel approach to Bayesian analysis, which is based on structural reliability concepts, termed BUS [1, 2]. By exploiting analogies between Bayesian analysis and the estimation of the probability of rare events (i.e. classical structural reliability problems), BUS offers a plethora of possible solution strategies. In particular, the authors have applied Subset Simulation [3], which has shown to enable efficient and robust Bayesian analysis in high dimensions [4]. However, other structural reliability methods can be applied and are more efficient for certain types of (lower-dimensional) problems. For example, the First-Order Reliability Method (FORM) or importance sampling methods such as line sampling are employed in [1].

Because BUS is based on structural reliability methods, it is particularly effective for updating the probability of rare events. Other approaches to Bayesian updating of rare events rely on a two-step approach, by first computing samples from the posterior distribution, and then estimating the probability of the event based on these samples, e.g. [5]. For rare events, the probability of interest is low, and a two-step approach is computationally expensive. On the other hand, the BUS approach allows to solve the Bayesian updating problem in a single step, by formulating the Bayesian updating as a system reliability problem. This allows using sampling approaches that are optimized towards solving such a system problem, but it also enables the use of non-sampling based approaches, in particular FORM/SORM methods.

In this contribution, the application of FORM-based BUS approaches to solving the Bayesian updating of rare events is investigated in detail. The factors influencing the performance of the algorithm are studied and recommendations on the use of the method are made. The algorithms are applied to illustrative examples as well as to a realistic large-scale example.

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# Polynomial chaos approach for tuned bladed rotors with geometric modification

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Bladed rotors are often considered in theory as periodic systems, but in fact they feature inevitable small imperfections caused by material defects and manufacturing tolerances which break the cyclic periodicity. This is called mistuning. The loss of the periodicity changes drastically the dynamic behavior of a rotor. Typically the forced response level of the mistuned bladed rotor is larger than the tuned design. Because of the random nature of mistuning, the determination of this amplitude magnification has to be considered as a stochastic problem. One of the classical ways to analyse mistuning is then to perform a Monte Carlo simulation.

In order to minimize the computational costs, a reduced order model has to be generated from the finite element model of the rotors. The way in which mistuning is implemented depends on the used reduction method. Recently Sinha [1, 2] developed a method to consider geometric mistuning via random variations in geometries of blades. This method has been called modified modal domain approach (MMDA). In the same time Mbaye et al. [3] developed another reduction method for intentionally geometric mistuning. This method is based on the use of the cyclic modes of the different sectors, which can be obtained from a usual cyclic symmetry modal analysis.

We search to create a new method inspired of this both methods. The key idea is to use a polynomial chaos approach to approximate the cyclic modes of the different sectors with random geometry modifications. Let us consider the rotor with all blades having the same random geometry. The cyclic modes are solution of the following stochastic problem:

$$[-\omega_k^2(\boldsymbol{\xi}) \mathbf{M}(\boldsymbol{\xi}) + \mathbf{K}(\boldsymbol{\xi})] \boldsymbol{\psi}_k(\boldsymbol{\xi}) = \mathbf{0}$$

$\mathbf{M}$  and  $\mathbf{K}$  are the mass and the stiffness matrices and  $\boldsymbol{\xi}$  is the standard normal random vector of geometric variations. The eigenvalue  $\omega_k^2$  and the associated eigenvector  $\boldsymbol{\psi}_k$  are considered respectively as a second-order random variable and a second-order random vector. They can then be approximated by a truncated polynomial chaos expansion. When the input random vector  $\boldsymbol{\xi}$  is standard normal, the adapted decomposition basis may be the family  $\{H_\alpha\}_{|\alpha|=0}^r$  of the  $p$ -dimensional Hermite polynomials of degree not exceeding  $r$ , which are orthogonal with respect to the Gaussian measure. Eigenvalues and eigenvectors are then approximated as follows:

$$\omega_k^2(\boldsymbol{\xi}) \simeq \sum_{\alpha, |\alpha|=0}^r f_{\alpha,k} H_\alpha(\boldsymbol{\xi}), \quad \boldsymbol{\psi}_k(\boldsymbol{\xi}) \simeq \sum_{\alpha, |\alpha|=0}^r \mathbf{g}_{\alpha,k} H_\alpha(\boldsymbol{\xi})$$

The computation of the chaos coefficients  $f_{\alpha,k}$  and  $\mathbf{g}_{\alpha,k}$  can be performed using a projection method based on Gaussian quadrature scheme.

This approximation might make it possible to built an appropriate projection basis for any random geometric mistuned bladed disk. The purpose of this contribution is to examine the accuracy of such approximation.

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## Damage models described by uncertain parameters

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Heterogeneities appearing on fine scales and incomplete knowledge of parameters governing the behaviour of continuum materials necessitate to consider the parameters defining their constitutive model as uncertain. Thus, the material parameters describing both the elastic/reversible behavior as well as inelastic/irreversible behavior are modelled as random variables/fields. To be able to simulate inhomogeneous and anisotropic behavior, the parameters of the model are random tensor fields with appropriate properties, like e.g. positive definitiveness.

In this work the focus is on the quantification of the uncertainty in the response of coupled elasto-plastic damage model described by material uncertainty. The uncertainties will be represented by Karhunen-Loève and polynomial chaos expansions, where particular care has to be taken to preserve the afore mentioned properties. The solution is computed with a non-intrusive stochastic Galerkin method.

Results are shown which show the influence of the modelling choices.



# Uncertainty quantification for linear elastic bodies with fluctuating input parameters

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In many physical and engineering applications deterministic solvers like the finite element method (FEM) are commonly used to treat problems which are described by partial differential equations (PDEs). Technically, many system parameters are stochastic. Thus, the deterministic PDE becomes a stochastic partial differential equation (SPDE). One possible approach to solve SPDEs is the stochastic finite element method (SFEM).

The governing equations of elasticity including the variational formulation of the SPDE are introduced. For the numerical application it is necessary to discretize the equation in the spatial and the stochastic domain. The spatial discretization is performed by ordinary finite element methods whereas the stochastic discretization uses WIENER'S polynomial chaos [2] (with so-called *Hermite polynomials* [1],[2] as basis functions) to expand the coefficients in deterministic and stochastic parts.

In this work a linear elastic body with two independent stochastic material parameters based on experimental data is studied. The key idea is the split of the elasticity matrix  $\underline{C}(\omega)$  into a deviatoric part  $\underline{C}_{dev}(\omega)$  and a volumetric part  $\underline{C}_{vol}(\omega)$ .

In order to compare the appropriate choice of fluctuating input parameters two different approaches are presented: The first one presupposes the knowledge of the distribution of the random variables *Young's modulus*  $E(\omega)$  and *Poisson's ratio*  $\nu(\omega)$ . The second one assumes the distribution of *shear modulus*  $G(\omega)$  instead of  $\nu(\omega)$  to be known. Computational approaches involving polynomial chaos are used to expand these variables. Therefore, GALERKIN projection [1] can be applied to reduce the stochastic PDE into a system of deterministic PDEs. In this work normally distributed random variables are considered. Thus, the number of stochastic dimensions is equal to the number of independent input parameters. Subsequently, the resulting equation system is solved iteratively. Finally, our method is applied in a numerical example for a plate with a ring hole.

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# Probabilistic Sensitivities for Fatigue Using Adjoint Methods

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We present a new approach to develop methods and design tools to determine the sensitivity of the probability of failure of mechanically and thermally loaded hot gas component  $\Omega$  under the variation of design parameters and production-related deviations. Based on the local and probabilistic model described in [1] and [2], we compute the sensitivity of the probabilistic cost functional  $J_{sur}(\Omega, u)$ :

$$J_{sur}(\Omega, u) = \int_{\partial\Omega} \left( \frac{1}{N_{det}(T, \epsilon(\nabla u(x)))} \right)^m dA$$

where  $u$  is the displacement,  $m$  is the shape parameter of the Weibull distribution and  $N_{det}$  is the Weibull scale parameter which is supposed to be depended on the elastic strain tensor  $\epsilon$  and the temperature  $T$ . This approach applied the adjoint methods to reduce the computational cost by computing these sensitivities.

$$\left\{ \begin{array}{l} \text{solve } \overbrace{\left( \frac{\partial R_M}{\partial u} \right)^T \lambda_1 = \left( \frac{\partial J}{\partial u} \right)^T}^{\text{purely mechanical adjoint equation}} \\ \text{and } \overbrace{\left( \frac{\partial R_T}{\partial T} \right)^T \lambda_2 = \left( \lambda_1^T \frac{\partial R_M}{\partial T} - \frac{\partial J}{\partial T} \right)^T}^{\text{thermal-mechanical adjoint equation}} \\ \text{and compute } \underbrace{\frac{dJ}{dX} = \lambda_2^T \frac{\partial R_T}{\partial X} - \lambda_1^T \frac{\partial R_M}{\partial X} + \frac{\partial J}{\partial X}}_{\text{global sensitivity equation}} \end{array} \right.$$

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# Irreversible material behaviour under presence of uncertainty

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Within the framework of small and large strain analysis the focus is set on the rate-independent evolutionary problem with general hardening whose material characteristics are assumed to be uncertain. By introducing the material properties in the form of random fields the irreversible and work-dissipating process is described via a stochastic convex energy function and evolution equations for internal variables. This allows the reformulation of the problem into the stochastic minimisation of a smooth convex energy functional on discrete tensor product subspaces whose unique minimizer is obtained via a stochastic closest point projection algorithm. Its numerical computation is performed with the help of methods of functional approximation, i.e. a direct, purely algebraic way of computing the response in each iteration of Newton-like methods. Furthermore, the method is contrasted to the less efficient but more accurate non-intrusive variant which evaluates the residuum in each iteration via high-dimensional integration rules based on random or deterministic sampling - Monte Carlo and related techniques. In addition to these, the problem is solved with the help of the stochastic collocation method. Finally, the methods are validated on a series of test examples whose reference solution is computed via direct integration methods.

# Jump Phenomena and Bifurcations in Stochastic Vehicle-Road Dynamics

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Vehicles riding on uneven rough road surfaces [1] are excited to vertical and horizontal vibrations. The describing system equations are linear when the vehicle velocity is constant and there is a control providing a stationary driving force process needed to maintain an exactly constant vehicle speed.

More important is the inverse problem of a constant driving force meanwhile the velocity process is stationary fluctuating around a mean velocity as the consequence of the up and down of roads. The paper investigates second order road profiles modelled by linear filter equations [2] under white noise which include the limiting profile of harmonic wave roads.

Because of the forced velocity fluctuations, the describing equations of motion are now highly non-linear effecting velocity jumps when the driving force reaches critical values. These phenomena are similar to the amplitude jumps of the stochastic Duffing oscillator which are possible for narrow-banded base excitations by roads.

The applied simulation and statistical evaluation [3] are explained in detail by means of the extended Kramers oscillator under white noise. The stationary solution of its Fokker-Planck equation can explicitly be derived showing typical jump phenomena in form of two-modular density distributions which are numerically estimated by means of a separation technique.

## References

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# Regional Frequency Analysis of extreme rainfall events in Tuscany (Italy)

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The assessment of extreme hydrological events at sites characterized by short time series or where no data record exists, has been mainly obtained by regional models. Regional frequency analysis based on the index variable procedure is implemented to describe the annual maximum of rainfall depth of short durations in Tuscany region. The probability distribution TCEV - *Two Component Extreme Value* is used in the frame of the procedure for the parameters estimation based on a three levels hierarchical approach [1]. This methodology deal with the delineation of homogeneous regions, the identification of a robust regional frequency distribution, the assessment of the scale factor, i.e. the index rainfall. The data set includes the annual maximum of daily rainfall of 351 gauge stations with at least 30 years of records, in the period 1916–2012, and the extreme rainfalls of short duration, 1 hour and 3, 6, 12, 24 hours. Different subdivisions hypotheses have been verified. A four regions subdivision, coincident with four subregions, which takes into account the orography, the geomorphological and climatic peculiarities of the Tuscany region, has been adopted. Particularly, for testing the regional homogeneity, the cumulate frequency distributions of the observed skewness and variation coefficients of the recorded times series, are compared with the theoretical frequency distribution obtained through a Monte Carlo technique. The related L-skewness and L-variation coefficients are also examined [2]. The application of the Student *t*-test and the Wilcoxon test for the mean, as well as the  $\chi^2$  was also performed. Further tests of subdivision hypotheses have been made through the application of discordancy *D* and heterogeneity *H* tests and the analysis of the observed and theoretical TCEV model growth curves. For each region the daily rainfall growth curve has been estimated. The growth curves for the hourly duration have been estimated when the daily rainfall growth curve is not able to describe the sample cumulate frequency distribution. In order to define the index rainfall for each homogenous region and each duration, a multivariate model that combines the Mean Annual Precipitation, the elevation, the aspect and the sample mean of the annual maxima time series of the gauge sites has been used.

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# On Gaussian approximation of the strength of Daniels' bundle with brittle Weibull fibers

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The paper deals with the classical fiber bundle model with equal load sharing (load of the broken filament is distributed uniformly non-broken filaments), sometimes referred to as the Daniels bundle [1] or the democratic bundle [4]. This model is significant for strength of quasi-brittle structures and reliability of many parallel systems. Daniels formulated a multidimensional integral and also recursive formula for evaluation of the strength distribution function: In the same paper, he showed the strength of the bundle,  $G_n$ , (the peak force) tends to Gaussian distribution under quite broad conditions and he gave closed formulas for the mean value and standard deviation of the Gaussian distribution. Sornette later confirmed this result using a Kolmogorov theorem. The convergence of a random peak force to Gaussian distribution is very slow in terms of the number of fibers and therefore, Smith [3] proposed a correction term for the mean value that improves the original Daniels formula for small bundles.

In the present paper, the authors exploit their own implementation of the recursive formula for evaluation of the distribution function for strength of the fiber bundle. The implementation was made in high-level programming language Python using scientific packages – NumPy (scientific computing with arrays) and mpmath (library for real and complex floating-point arithmetic with arbitrary precision). This implementation enables to calculate values of cumulative distribution function (CDF) for large numbers (thousands) of fibers in bundle including values deep in the left tail of the distribution (probabilities  $10^{-600}$ ). This computer program has been used to accurately calculate the distribution functions  $G_n$ :

$$G_n(x) = P(Q_n^* \leq x) = \sum_{k=1}^n (-1)^{k+1} \binom{n}{k} [F(x)]^k G_{n-k} \left( \frac{nx}{n-k} \right), \quad (1)$$

where  $G_1(x) \equiv F(x)$ ,  $G_0(x) \equiv 1$  and  $\binom{n}{k} = \frac{n!}{(n-k)! k!}$

for bundles with Weibull fibers  $F(x) = 1 - e^{-(x/s)^m}$  with scale parameter  $s = 1$  and varied number of filaments  $n$  and varied shape parameter  $m$  and results were stored in database. This database has been used to calculate the mean values and standard deviations of the peak force. These values are compared to the available formulas [1, 2, 3]. We have found that for small bundles ( $n$  between 2 and 100) the formulas are not accurate. Therefore the authors propose improved closed-form predictions for the mean and standard deviation.

The main motivation of the work is to formulate an improved analytical formula for the distribution function  $G_n$  that will be valid deep to the left tail where the real distribution strongly deviates from the Gaussian approximation.

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# Best practice in metamodeling for data derived from civil engineering applications

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In many fields of civil engineering, problems frequently exist in which a high computation time for determining the optimal deployment of construction materials is necessary. To reduce the costs in these situations and to keep the computation time low without losing model quality, the application of mathematical models is useful. With response surface methods, which are also called metamodels, calculations and forecasts can be completed faster because of a reduced amount of numerical or experimental data. In this contribution, the applicability of metamodels to civil engineering problems are first analyzed. Furthermore, it seeks to answer the question of which model is the most suitable for specific application fields.

The available types of metamodels can be divided into deterministic and stochastic models, whereby these groups differ significantly in their properties and thus in their applicability. For the deterministic methods, *Polynomial Regression* [2], *Radial Basis Function Interpolation* [2] and *Moving Least Squares* [5] are specific examples. In the case of the stochastic methods the *Stochastic Process Model (Kriging)* and its variations [1] are to be emphasized. In addition, all of the existing models differ greatly from each other regarding computational effort, flexibility, predictive ability and robustness. Moreover, there is a differentiation between interpolation and regression possible. Regressive methods are often more appropriate for noisy measurement data because a smoothing occurs.

In order to investigate when an application is useful, a specific example is considered, namely a variance based sensitivity analysis [6] of a model representing a triaxial soil test, in which various finite element methods are used. To achieve sufficiently accurate and robust values, a large amount of data is necessary, which magnifies the computation time with increasing number of input parameters. This can lead already for a comparatively simple model, Mohr Coulomb, with five parameters to a computation time of at least 22 hours.

This duration shows clearly that an application of metamodels could be extremely rewarding. By using for example linear regression, the computation time is reduced extremely and the solution shows a similar trend. However, there are also some questions that occur:

- How much data is necessary to produce the various metamodels?
- Will a less complex model be preferred despite a lower accuracy?
- And, according to what criteria is the decision made, which model is optimal?

At that, it is always a trade-off between expense and accuracy by which the decision gets more complicated and may vary depending on the situation. Because of that, there are different error calculations to compare the various metamodels. It must be distinguished whether an untrained additional data set is available to test the quality of the response surfaces and the prediction ability or whether the existing data is completely used for the model construction. The complexity of the model, which depends on the number of unknown model parameters, can also be included in this analysis. Different kinds of error values and comparison criteria are described for example in [3] and [4].

The aim for the analysis of response surface methods with several examples, like the mentioned sensitivity analysis, is to infer to the general applicability for models used in civil engineering. In this context it is necessary to classify the engineering models based on their underlying partial differential equation, which can be elliptic, parabolic and hyperbolic. This separation is related to the different engineering fields of the problems. For example, static systems and problems related to the heat equation differ extremely. Because of that, it is probable that different metamodels are optimal for each application field. In each of these partial differential equation categories, fundamental solutions exist, which provide in the simplified one dimensional case explicit and simple to evaluate functions that can be used for tests.

A case that can also be important is one in which no single metamodel has emerged as the optimal method, so that it is necessary to discuss a combination of several response surface methods. In conclusion, it is possible to increase the use of metamodels by providing an overview of the relation between these models and the corresponding classes of civil engineering problems and thereby improve the construction of engineering models.

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# Asymptotic sampling - a tool for efficient reliability computation in high dimensions

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Advanced Monte Carlo methods developed over the past years allow the computation of very small exceedance probabilities such as those required for structural reliability analysis with high precision. Typically these methods focus on a particular fixed threshold value for which the exceedance probability is computed. It may, however, sometimes be more convenient to have information about the entire tail of the distribution rather than only one specific quantile. This is particularly useful in the context of reliability-based structural optimization. This contribution presents an extension of the asymptotic sampling method to compute the failure probability using a small number of samples with artificially increased standard deviations. Numerical examples demonstrate the applicability and efficacy of the suggested approach.

Generally, the probability of failure  $P_F$  in an  $n$ -dimensional space of random variables  $X_1, \dots, X_n$  can be computed as [3]

$$P_F = \int \cdots \int_{D_F} f_{X_1, \dots, X_n}(x_1, \dots, x_n) dx_1 \dots dx_n \quad (1)$$

In this equation,  $f_{X_1, \dots, X_n}(x_1, \dots, x_n)$  denotes the joint probability function of the random variables  $X_1, \dots, X_n$  and  $D_F$  denotes the failure domain, i.e. the region of the  $n$ -dimensional random variable space in which failure occurs. Typically, this is denoted in terms of a scalar limit state function  $g(\cdot)$  attaining negative values, i.e.  $D_F = \{(X_1, \dots, X_n) \mid g(X_1, \dots, X_n) \leq 0\}$ .

Since the failure probabilities to be computed are usually very small, say of the order of  $P_F = 10^{-5}$ , it is not feasible to evaluate the integral in Eq. (1) using standard numerical integration procedures. This is due to the fact that the number of integration points required to perform e.g. Gaussian integration in dimension  $n$  grows exponentially with  $n$ . Monte Carlo methods do not have this dependence on the dimension, however, crude or plain Monte Carlo simulation requires a number of samples roughly larger effort. Hence, specific methods have been developed which utilize the fact that in many cases only a small region in  $n$ -dimensional space actually contributes significantly to the failure probability. Sampling methods based on this idea are called importance sampling [4]. For good efficiency, these importance sampling methods require some level of detailed knowledge about the problem, and in many cases this knowledge is primarily based on the most probable failure point or design point. If such information is not available, or can be made available only at high computational expense, alternative methods such as Asymptotic sampling [2] may be advantageous. This method does not rely on the knowledge of the relevant failure region, but rather on the scaling of the failure probability (or reliability index  $\beta$ ) with a scaling of the standard deviations in uncorrelated Gaussian space. Asymptotic properties of this relation [1] are exploited.

The contribution will review this method in the light of its applicability to systems reliability analysis and provide new insights into the optimal choice of algorithmic parameters.

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# Polynomial Chaos and the Heave Motion of a Cylinder in Random Seas

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In this contribution, the random heave motion of a floating cylinder subject to stochastic wave forcing is investigated numerically. In particular, we obtain numerical solutions for the response distribution with the generalized Polynomial Chaos (gPC) framework for irregular wave excitation described by the JONSWAP spectrum.

Floating offshore systems are generally subject to irregular sea waves, which can be described by sea spectra such as the JONSWAP or the Pierson-Moskowitz spectrum. Due to the stochastic forcing, the responses of the considered systems are stochastic processes as well. Within the framework of stochastic dynamics we are dealing with stochastic equations of motion excited by colored noise. In offshore engineering, the frequency domain approach is usually the method of choice for obtaining solutions when the considered systems are assumed to be linear. However, the analyses of transients and nonlinear systems require time domain methods.

The direct Monte Carlo method is the simplest method for treating stochastic ordinary differential equations in time domain and is widely applied throughout the ocean engineering community. It is well known that the accuracy of the solution obtained with the direct Monte Carlo method scales inversely to the square root of the number of samples, e. g. in general the Monte Carlo method is computationally expensive if a high accuracy is desired. Xiu and Karniadakis [1] present a method for solving stochastic differential equations, popularly known as the generalized Polynomial Chaos framework. Numerical examples show a substantial speed-up compared to the direct Monte Carlo method. Although many parts of the scientific computing community have taken great interest in the gPC framework in the recent years, there are not many applications to problems in offshore engineering so far.

The analysis of the heave motion excited by irregular waves requires two steps within the gPC framework. First, the random forcing has to be represented as a truncated Karhunen-Loève (K-L) expansion. This representation of the stochastic waves in terms of a few uncorrelated random variables can be obtained by solving a Fredholm integral equation of the second kind. Thereby, the kernel can be obtained from the power spectral density of the sea state, which is generally known (see above). The solution of the Fredholm integral equation allows to state the stochastic process as a linear superposition of eigenfunctions with a minimum number of stochastic dimensions. For narrow banded sea states only very few terms in the K-L representation are required indeed. Despite its importance, the implementation of the K-L is considered as a complex task, because of the difficulty in solving the Fredholm integral equation. However, Sclavounos presents in a recent work [2] a new representation of stochastic ocean waves based on the K-L decomposition.

The second and final step in order to determine the response of a stochastically excited offshore structure is to transform the stochastic equation of motion in deterministic ordinary differential equations. The unknown stochastic solution of the equation of motion is stated with a polynomial chaos expansion (PCE) ansatz. The theoretical justification derives from the Cameron-Martin theorem, which states that the PCE of a stochastic process converges in  $L_2$  to the stochastic process itself. This solution ansatz is substituted into the equation of motion. We take a Galerkin projection of the equation of motion onto previously chosen basis polynomials. The procedure ensures that the stochastic equation of motion reduces to a system of coupled deterministic differential equations for the coefficients of the PCE. Once the PCE coefficients have been computed with ordinary time integration schemes, the approximate moments of the distribution of interest can be obtained.

We present the application of the method to the example of a heaving cylinder in irregular waves.

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# Stochastic quantification of the impact of uncertainty in inlet conditions on the aerodynamics of a 5:1 rectangular cylinder

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The Benchmark on the Aerodynamics of a Rectangular Cylinder (BARC) collects the results of wind-tunnel experiments and of numerical simulations of the flow around a rectangular cylinder with chord-to-depth ratio equal to 5 and infinite spanwise size. The considered flow configuration is of practical interest, e.g. in civil engineering, and, in spite of the simple geometry, the related flow dynamics and topology is complex. The results of about 70 experimental and numerical realizations of the BARC flow configuration were collected and reviewed in [1]. It was observed that some of the flow quantities, as the standard deviation of the lift coefficient, the distribution of mean and fluctuating pressure on the cylinder sides are affected by significant dispersion, both in experiments and in simulations. The observed dispersion can be explained by the extreme sensitivities of some flow features to uncertainties which can be hardly avoided or controlled in experiments or simulations. In particular, besides modeling uncertainties and numerical errors, in numerical simulations it is difficult to exactly reproduce the experimental conditions due to uncertainties in the experimental set-up parameters. We investigate herein the impact of some parameter uncertainties on the numerical predictions of the BARC test case using probabilistic methods. The following uncertain set-up parameters are investigated in the subsequent uniform ranges: the angle of incidence  $\alpha$  ( $-1^\circ - 1^\circ$ ), the longitudinal turbulence intensity  $I_x$  (0.001 – 0.03), and the turbulence length scale  $L$  ( $0.1D - 5D$ ),  $D$  being the cylinder depth. These are quantities which are difficult to exactly control and measure in experiments. Numerical simulations are carried out by means of a URANS approach. The Stochastic Collocation (SC) method [2] is employed to perform the probabilistic uncertainty propagation of the three set-up parameters. This results in 25 URANS simulations based on the Smolyak sparse grid extension of the level-2 Clenshaw-Curtis quadrature points. The propagation of the considered uncertainties on the quantities of interest for the BARC test case [1] is then characterized in a probabilistic manner. The numerical error is estimated by carrying out the stochastic analysis on two different grids. The impact of turbulence modeling is similarly investigated by repeating the analysis for two RANS closure models. This additional uncertainty quantification information can lead to new insights into the unresolved issues of the BARC flow test case.

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# A Bayesian approach for the quantification of parametric and model-form uncertainties in dense-gas flow models

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In this work, we adopt a Bayesian approach to quantify uncertainties associated to the thermodynamic models used for the simulation of dense gas flows, i.e. flows of gases characterized by complex molecules in thermodynamic conditions of the general order of magnitude of the liquid/vapor critical point. Thermodynamic data about the working fluid in use are used for calibrating the material-dependent coefficients of equations of state (EOS) used to model the thermodynamic behavior of the working fluid. It has been shown that dense gas flow simulations can be extremely sensitive to the model used to describe the fluid thermodynamic behavior [1]. EOS give raise to two kinds of uncertainties: the first one concerns the mathematical form of the EOS to be used for a given fluid; on the other hand, the material-dependent coefficients associated to the EOS are imperfectly known. Previous work [1] has shown that for some particularly complex gases the model-form uncertainty can be even overwhelming with respect to the parametric uncertainty. This is why we look here for a methodology allowing to account simultaneously for both forms of uncertainty.

As a first step, the available thermodynamic data on the fluid of interest are used for the Bayesian calibration of the material-dependent coefficients of a EOS. This results in posterior distributions of the coefficients than can be propagated through the flow solver to quantify the uncertainty on the output quantities of interest (QoI) and compared to aerodynamic data available for validation purposes. Different popular EOS for dense gases, like the cubic Soave-Redlich-Kwong [2] and Peng-Robinson-Strjyek-Vera [3] EOS and the 5th-order virial Martn-Hou EOS [4], are calibrated using either pressure-volume-temperature data and/or speed of sound data. The statistical model includes both an experimental error term and a model-inadequacy term. The results are then propagated through a numerical dense gas flow solver [5] to quantify the resulting uncertainty on output quantities of interest like the pressure distribution at the airfoil wall or the aerodynamic coefficients. For this purpose we use a response-surface method which reconstructs the code response to the thermodynamic model coefficients as an analytical function and reduce the computational cost (see, e.g. [6]). The flow configuration considered here is a transonic flow over an airfoil. The simulations are carried out for different choices of the operating conditions, and namely of the thermodynamic state, Mach number and angle of attack of the incoming flow, to investigate the predictive capabilities of the calibrated models.

Secondly, to account for the availability of several alternative models, a Bayesian-model-average of the three EOS under study is constructed. The averaged equation, which accounts simultaneously for parametric and model-form uncertainty, is also propagated through the dense gas code and the improvements over the standard approach based on a single model structure are investigated.

Full descriptions of the statistical methodology and of the results will be provided at the conference.

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# Investigation of cavitation and turbulence models uncertainties for cavitating flows

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The collapse of vapour bubbles in high-pressure region can produce dramatic effects, such as failure, erosion and other undesirable effects. An open question is still related to the interaction between cavitation and turbulence. In particular, an accurate description of interactions between the vapour and liquid phases requires accurate physical models and a way to take into account the dynamics of the interface, multiscale effects, turbulence and thermodynamics.

Several numerical approaches have been proposed to reproduce cavitating flows in external and internal configurations. Principally the models can be regrouped in two major categories: interface models and two-phase models. In the first case, the liquid and the vapour phase are separated by an interface, then the systematic reconstruction of interface and the applicability to complex geometries are the most challenging issues. Concerning two-phase models, the two phases are treated as a mixture. Difficulties of these models are related to the mixture's properties estimation based on the liquid-vapour mixture ratios [1]. Differences between the various models in the second category mostly come from the relation that defines the density field. For more details concerning the various modelling approaches, Refs. [3, 4, 2] are strongly recommended.

A large literature exist on the use of RANS model for performing this kind of computation. Nevertheless, the assumption of some empirical coefficients remains somehow arbitrary, and sometimes tuned for reducing the distance between experiments and numerical solution. For these reasons, it is crucial to consider this epistemic (since it is due to a lack of knowledge) uncertainty in the problem, thus providing a measure of the variability of the numerical solution, *i.e.* assessing the quality of the numerical prediction. Recently, Rodio & Congedo [5] have proposed a study about the impact of various sources of uncertainty (on the cavitation model and on the inlet conditions) on the prediction of cavitating flows by coupling a non-intrusive Polynomial Chaos stochastic method with a cavitating CFD solver.

This work is focused on the assessment of different sources of uncertainties for a Venturi configuration, which is one of the most studied for cavitating flows. In particular, contributions are related to the propagation of uncertainties for some turbulence model parameters and their calibration with respect to the experimental data.

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# Influence of Uncertainties of the Aerodynamical Parameters on the Simulation Model for a Civil Aircraft with Active High Lift System

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This contribution presents the propagation of uncertainties of the eigenmotions of an active high lift civil aircraft model. These uncertainties are due to incertitude of the aerodynamical parameters of the model.

With the aim to extend the usage of already existing small airports in the close vicinity of bigger cities, a new medium range civil aircraft design, that can take off and land on relatively short runways, was proposed [1]. The short take off and land distance is achieved by an active high lift system based on an internally blown flap system. However, among other design difficulties, extreme high lift along with low dynamic pressure puts high demands on the trim and controllability of such an aircraft. In addition there is a strong cross-coupling between lift and thrust resulting in the need for active control to achieve acceptable handling qualities [2]. In order to be able to test maneuverability of the aircraft a stochastic aerodynamic analysis was carried out. First, a nonlinear deterministic model was developed by the Institute for Flight System at DLR, simulating the longitudinal motion of the active high lift supported aircraft [2].

The main focus of this contribution is to enhance the reliability of this simulation model by quantifying its uncertainties, that is to analyse the aircraft behavior under the influence of uncertainties of its aerodynamical coefficients. This quantification was carried out in a non-intrusive manner, meaning that the original model was treated in a black box fashion, while the constraints and input parameters were changed to random variables instead of deterministic ones. The probability distribution function of the uncertain parameters were defined by the professionals. Besides analysis of the aircraft model the contribution targets to present different non-intrusive approaches, with which such sensitivity analysis and uncertainty quantification can be carried out without significantly changing the original, deterministic simulation model. The non-intrusive uncertainty quantification was first developed for a basic aircraft simulation model [3] and further extended in this contribution for the targeted updated aircraft simulation model, that already includes the active high-lift system.

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# Stochastic quantification of the effects of inlet velocity conditions on the dynamics of spatially evolving mixing layers

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Mixing layers are present in many technological applications or environmental flows and they have been extensively studied in the literature. Mixing layers are characterized by the evolution and the interaction of coherent vortex structures, which form at the interface between two parallel coflowing streams with different velocities and play a major role in the bulk mixing of the fluids and in the growth of the mixing layer. It is well known that the inlet conditions, which may be not exactly known in practical applications, significantly affect the dynamics of mixing layers. The most common assumption in studying this flow configuration is that the two coflowing streams are parallel, i.e. the two inlet bulk flow velocities have the same direction; in this case, the coherent vortex structures form due to the effect of the gradient of velocity occurring in the normal direction. However, it may happen in practice that this condition is not verified and that the two streams are actually tilted of an angle  $\theta$ . Previous experimental studies [1] have shown that the flow evolution is extremely sensitive to the three-dimensionality of the inlet velocity field. A second parameter influencing the flow transition from the laminar regime to the fully developed turbulent one, as well as the dynamics of the generated eddies [2], is  $\alpha = (U_1 - U_2)/(U_1 + U_2)$ , where  $U_1$  and  $U_2$  are the two asymptotic stream velocities.

In the present work, we propose to quantify the impact on flow evolution of the angle between the two streams,  $\theta$  and of  $\alpha$  through direct numerical simulations and by adopting a stochastic approach, in which  $\theta$  and  $\alpha$  are considered as aleatory variables and the uncertainty is propagated over the output quantities of interest. To reduce the required computational resources, a continuous representation of the investigated flow quantities over the random variable space (response surface) is obtained through a simple, inexpensive model. The response surface is reconstructed starting from a limited number of samples taken at known discrete points on the uncertainty space, i.e., single deterministic numerical simulations. In particular, as already done for classical mixing-layers in [3], the response surface is built through generalized Polynomial Chaos (gPC).

The deterministic simulations needed to obtain the response surface are direct numerical simulations carried out through the open-source code OpenFOAM. The Reynolds number is  $Re = \frac{(U_1 - U_2)\delta_0}{2\nu}$  is set equal to 100, in which  $\delta_0$  is the inlet vorticity thickness and  $\nu$  the kinematic viscosity. The following ranges of variation of the random variables have been chosen:  $0 \leq \theta \leq \pi/12$  and  $0.2 \leq \alpha \leq 0.7$ . As previously mentioned, in our approach, each deterministic simulation corresponds to a discrete point in the two dimensional uncertainty space. These points are the quadrature nodes of a two dimensional Gauss-Legendre quadrature of the uncertainty space. This quadrature rule has been chosen as it accurately fits with the enforced probability density function of the random variables, which has been chosen as uniform. The gPC expansion for the two random variables is truncated at the third order, which means that the minimum number of simulations required for a full convergence of the response surface model is  $4^2$ . The flow quantities of interest, for which the uncertainty propagation is quantified through the response surface, are the three components of the mean velocity vector,  $U$  and of its quadratic function  $M = (U_1 - U)(U - U_2)$ , which is related to the momentum thickness. The spatial evolution of the two quantities is analyzed for each component as well as for the vector magnitude. The results are as well compared with the information which could be obtained by deterministic DNS simulations, carried out for different values of the input parameters.

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# Solution of the First passage problem by Path Integration for normal, Poissonian, and alpha-stable white noise

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In this contribution, the first passage problem is examined for linear and nonlinear systems driven by Poissonian and normal white noise input. The problem is handled step-by-step accounting for the Markov properties of the response process and then by Chapman-Kolmogorov equation. The final formulation consists just of a sequence of matrix-vector multiplications giving the reliability density function at any time instant. Comparison with Monte Carlo simulation reveals the excellent accuracy of the proposed method.

The first passage problem has been investigated in many publications over almost a century because of its relationship to the safety of structural systems under random excitations. The exact solution to the first passage problem is not available because even in the case of a normal white noise process the Fokker-Planck equation with associated boundary conditions is in general unknown [11]. Approximate methods have been proposed ([2, 8, 1, 12, 10]), however, the analytical approximation methods are available only for light damping for the stochastic averaging, weak nonlinearity and Gaussian approximations.

In order to determine the probability distribution of the first passage time, efficient solution of the Fokker-Planck equation is necessary. Moreover, we need a solution of the problem step-by-step in order to cancel the trajectories that for the first time leave the safe domain (absorbing barrier problem). In order to have such a control on the path of the trajectories the only way is using the so-called *Path Integration* (PI) method. It mainly consists of using the Chapman-Kolmogorov (CK) equation giving the probability density at a certain time instant as weighted sum of the contributions of the various trajectories that in a previous time instant start with deterministic initial condition. As the interval between the two time instants becomes small, then the so-called *short time Gaussian approximation* [9] remains still valid and the step-by-step solution technique of the CK equation reverts to the PI method. Many papers have been devoted to this subject for normal [5, 13, 3] and Poissonian white noise [6, 7].

The PI method is versatile and in [4] it has been used for solving the first passage problem. It mainly consists in defining the so-called *reliability function* which is a function giving the probability that the various trajectories will remain inside the safe barrier conditioned by the fact that each of them never crosses the barrier up to the observation time.

In this contribution, by using the concepts exploited in [4] the first passage problem is revisited in the light of the cell mapping method and extended to the case of Poissonian and alpha-stable input. It is shown that the reliability function by discretization of the Chapman-Kolmogorov equation may be easily implemented in a computer program as just a sequence of matrix-vector multiplications whose sizes depend on the threshold barriers and the spatial discretization steps. Moreover, as the input is stationary the reliability function is ruled by a transition matrix that does not explicitly depend on time so that it can be computed once beforehand.

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# Adaptation and Enhancement of Generalized Polynomial Chaos for Industrial Applications

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Many industrial applications involve uncertain data. Quantifying them is a big challenge. For example, unknown disturbances can influence the parameters of the system, and thereby the result of the simulation. Measurements have to be performed when comparing the model with the reality, but sensors are also noisy. It is, in some cases, not even possible to state an exact model because of insufficient knowledge of the system. Addressing all those uncertainties simulation models need to be expanded by stochastic elements. Which leads us to the requirement that we should be able to numerically compute a stochastic solution.

This contribution presents ideas, how to adapt stochastic elements to industrial applications and reduce the number of simulations in comparison to Monte Carlo. Especially when the mathematical model equations are not known and just an input/output interface is given. For this so called black box model which can be generated by many simulation tools the generalized polynomial chaos based on the idea of Wiener [2], and extended by Xiu and Karniadakis [3] is a suitable technique. One main advantage of this concept can be seen in the ability to produce functional representations of stochastic variability. This yields in a reduction of the simulation number compared to classical methods and can be intensified by using a sparse grids approach [1].

Looking at industrial applications the concept can be applied to a huge amount of systems. For example, components including electronic drives, fuel injection valves, and mechanical parts. As an example of use a simplified model of a DC drive with six uncertain parameters is taken into account.

## References

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