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Scientific Program - Timetable

Sunday 22	Time	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
	9:15		Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
	9:30					
	9:45					
	10:15	Registration		von Mises prize lecture		
	10:30					
	10:45					
	11:15		Coffee Break	Coffee Break	Coffee Break	Coffee Break
	11:30					
	11:45					
	12:15		Plenary Lecture Thomas Böhlke	General Assembly	Plenary Lecture Ferdinando Auricchio	Contributed sessions (11 in parallel)
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	13:15	Opening	Lunch	Lunch	Lunch	
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	13:45	Univ. Chorus Performance				Closing
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Registration pre-opening	16:15	Coffee Break	Coffee Break Poster session	Coffee Break	Coffee Break Poster session	
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	16:45	Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Contributed sessions (14 in parallel)	Contributed sessions (15 in parallel)	Contributed sessions (15 in parallel)	
	17:15					
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S03: Damage and fracture mechanics

The section focuses on damage mechanisms and fracture mechanics for all kinds of solid materials and structures. It aims at bringing together related research including experimental observations, modeling approaches and numerical techniques. In general, material failure is a complex process which may be considered on different length scales ranging from the atomistic up to the component, or even to the structural, level. Besides the material behavior of course also aspects of loading situations are crucial to describe failure. Thus, contributions focusing on static failure, dynamic failure or cyclic fatigue are equally welcome.

Stabilizing the XFEM for static and dynamic crack simulations

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The eXtended Finite Element Method (XFEM) is a widely used technique for the simulation of cracks, heterogeneities and other problems in solid and fluid mechanics showing discontinuities in the field variables or their derivatives or localization phenomena of any type. It has shown to be very efficient and attractive for two and three dimensional simulations. However, the geometry of the discontinuity in conjunction with the mesh geometry and the applied XFEM enrichment scheme might in certain cases lead to very badly conditioned or even singular coefficient matrices, in general due to (near) linear dependencies between standard and enriched degrees of freedom. During the last years several stabilization techniques have been proposed in literature based on modifications of the enrichment functions, special preconditioning techniques, modifications of the mesh geometry or enrichment scheme or direct modifications of the resulting equation system.

In this contribution advantages and disadvantages of some of the stabilization techniques are presented. Additionally a simple but very efficient regularization method presented in [1] is combined with other stabilization techniques and extended to dynamic crack simulation problems. The technique is based on an eigenvalue decomposition of the element stiffness matrices of selected fully enriched elements and a stabilization of the subspace that causes ill-conditioning. This regularization technique has very little effect on the solution of the resulting equation system but significantly improves the condition number and is thus ideal especially for large three dimensional eXtended Finite Element problems. This is demonstrated with a number of numerical examples for static and dynamic crack simulations.

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Thermo-mechanical modeling of crack propagation in dynamically loaded elastomer specimens using a scaled boundary finite element approach

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Recently, a scaled boundary finite element (SBFE) formulation for geometrically and physically nonlinear materials has been developed [2] using the scaled boundary finite element method (SBFEM) [3]. The SBFE formulation has been employed to describe plane stress problems of notched and unnotched hyperelastic elastomer specimens. However, elastomers show an inelastic nonlinear mechanical behavior at large deformations which is temperature-dependent in general.

In this contribution, the derived SBFE formulation is extended to nonlinear time- and temperature-dependent material behavior. Subsequently, the SBFE formulation is incorporated into a crack propagation scheme to model crack propagation in cyclically loaded elastomer specimens. Crack propagation is supposed to start at an initial notch. As benchmark example, a pure shear specimen of the so-called tear fatigue analyzer (TFA) is addressed. A plane and initially notched elastomer specimen is subjected to force-controlled cyclic loading at different excitation frequencies. The simulation concept is characterized by A) a multigrid finite element computation for displacement and temperature field, B) treatment of the thermo-mechanically coupled problem in the form of subproblems to account for different time scales resulting from high excitation frequencies and slow-acting temperature increase, C) the evaluation of crack growth criteria and crack propagation direction criteria in the near field of the crack tip, D) a nonlinear finite viscoelasticity description of the elastomer material [1] for a wide frequency and temperature range, and E) a mesh modification procedure to represent crack growth. A curved crack path is tracked by mesh modification and remapping of the solution fields. The influence of the crack tip near field (heterogeneously distributed material properties, dynamic loading, viscous process zone, temperature state) on the resulting crack path and potential crack path deflections is numerically examined. In addition, the temperature increase at the crack tip and the rest of the elastomer specimen due to mechanical energy dissipation is evaluated and compared.

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Fitting stress intensity factors from crack opening displacements in 2D and 3D XFEM

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In the literature, a large number of methods exists which deal with the computation of stress intensity factors (SIFs). Many of these approaches are energy-based such as those based on the J- or interaction integral. A disadvantage of these methods is that the extension to 3D is difficult. An easier access is possible by techniques which observe the displacement or stress fields at the crack tip, e.g. displacement extrapolation or fitting approaches. These methods are intuitive and the extension from 2D to 3D is often straightforward. A comparison of these approaches is given by Muthu [4].

We propose the computation of SIFs by crack opening displacements (CODs) in linear elastic fracture mechanics (LEFM) with stressfree crack surfaces, in the context of the extended finite element method (XFEM) and a hybrid explicit-implicit crack description [1, 2, 3]. An explicit crack description by means of straight line segments in two dimensions or flat triangles in three dimensions has the advantage that the update of the crack geometry during crack propagation is quite simple. For calculations with the XFEM, an implicit crack description is used by means of level-set functions, which are derived from the explicit master configuration.

To compute SIFs by CODs, points have to be found on the implicitly described crack surface. As the displacement field is discontinuous on the crack surface the points have to be split in two opposite but infinitisimally close points. The global displacements of these points are extracted from the XFEM approximation. The difference of these two points describes the CODs in the global coordinate system. This CODs is now compared with the expected opening for a pure mode I, II and III. Therefore, a reference coordinate system has to be found, in which the analytical modes are evaluated. This reference system is aligned with the (curved) crack surface and extracted from the implicit level-set functions. With this special description of the local coordinate system no differences exist whether the crack is planar or not. The CODs for pure mode I, II and III are compared to the approximated CODs in order to fit SIFs. Another advantage of this approach is that mode III is consistently treated as the other modes.

The numerical studies show how to specify the fitting regions on the crack surface and confirm that accurate SIFs are robustly obtained with the proposed approach.

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On corner singularities in Reissner's theory of elastic plates

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In the framework of linear elasticity, stress singularities may occur in domains with non-smooth boundary like corners or sharp notches. In applications of fracture mechanics, the local stress field in the vicinity of stress concentrations is of particular interest. Therefore, methods of asymptotic analysis can be applied advantageously. The singularity analysis of elastic plates goes back to Williams [4] using the classical Kirchhoff-Love plate theory and a series expansion of the containing stress function. Extending the classical theory by taking shear deformations into account leads to Reissner's theory. The analysis of singularities for Reissner's plate theory has been examined by some authors using a stress potential approach and a Williams-type-expansion [1, 5] or a separation of variable ansatz and a series expansion for the displacements [2]. The solution obtained by a mathematical elaborate formulation based on weak solution theory in [3] indicates that the former results are incomplete.

In the present work, an asymptotic solution of the governing system of partial differential equation of Reissner's plate theory is obtained by using a complex potential approach. Within an asymptotic analysis, a reduced PDE-System of Reissner's theory is obtained, which can be solved analytically by introducing three arbitrary holomorphic potentials, which determine the deformations as well as the resultant forces and moments. This formulation allows for describing the singular behaviour of the stresses near a wedge vertex. Within a singularity analysis, the singularity exponent λ is calculated for different boundary conditions on the edges. To solve the boundary value problem, the three complex potentials have to be in a form so that the prescribed boundary conditions are fulfilled. By choosing an adequate ansatz for the holomorphic functions, this requirement leads to a characteristic equation, whose roots represent the singularity exponent. The obtained results coincide with those in [3] but the present approach can be applied with less mathematical effort.

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Crack propagation at bi-material interfaces

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The manner in which a crack propagates through a composite material significantly affects its macroscopic toughness. Consider, for example, a simple composite consisting of two materials bonded to each other. The first material contains a crack through its entire thickness and incident on the interface between the materials (the bond line). The crack can thus propagate in one of two ways; either along the interface (referred to as deflection) or across the interface and into the second material (referred to as penetration). Deflection leads to crack blunting and macroscopic toughening of the composite while penetration consists of direct propagation of the initial crack and macroscopic embrittlement.

Historically, the work of He and Hutchinson [1], based on linear elastic fracture mechanics (LEFM), has been widely used to determine deflection versus penetration behavior. Using this approach, a critical ratio of material toughness to interface toughness indicates when crack propagation transitions between deflection and penetration. However, more recently, the finite element work of Parmigiani and Thouless [2] has shown that the LEFM approach with its corresponding metric of only a critical toughness ratio is generally insufficient and that a combined strength and toughness approach is required. Using this combined approach, propagation by deflection versus penetration is determined by a critical ratio of material strength to interface strength, a critical ratio of material toughness to interface toughness, and several other dimensionless parameters. The finite element work of Strom and Parmigiani [3] provided further clarification by showing that the LEFM approach is only valid for conditions distant from those corresponding to the transition between deflection and penetration. Accurate predictions of penetration versus deflection behavior near transition conditions (which is typically the region of greatest interest) require the strength-and-toughness approach.

Currently published work using the strength-and-toughness approach focuses on comparisons to the LEFM approach, trends in toughness and strength ratios at transition, and applied loads at transition. The focus of the work presented here is to discuss how transition conditions vary for different specific materials. This is new information not included in prior published work. The results are applicable to a variety of material systems but are presented in terms of intergranular (i.e. deflection) versus transgranular (i.e. penetration) fracture of grains of a number of materials. Results show that delfection is more likely to occur as grain toughness increases , grain stiffness decreases, and grain size decreases. The phase angle (relative measure of mode I and mode II contributions to fracture) of the propagating crack is also considered and is shown to increase as grain toughness decreases, grain stiffness increases, and grain size increases. These results are significant in that they provide additional insight into how a crack incident on an interface will propagate in specific materials.

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Global approaches for an accurate loading analysis at multiple cracks systems

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Based on the work by Eshelby [1, 2], the path-independent J_k-integral [3, 4], M- and L-integrals [4, 5] and the interaction-integral [6] or I_k-integral were introduced and applied to cracks for the accurate calculation of crack tip loading quantities. Applying the FE-method to solve boundary value problems with cracks, numerically inaccurate values are observed within the crack tip region affecting the accuracy of local approaches. Simulating crack paths, local approaches face further problems as cracks are running towards interfaces, internal boundaries or other crack faces. Within global approaches, the path-independent integrals are calculated along remote contours far from the crack tip, essentially exploiting numerically reliable data requiring special treatment only for the near-tip crack faces [7, 8]. To provide path independence, additional integrals along interfaces, internal boundaries and crack faces are necessary.

J_k-, I_k-, M- and L-Integrals are akin to each other but bare different features which may be utilized for the development of global approaches considering multiple defects. A global approach of the I_k-integral can be used for the loading analysis of multiple cracks, by using appropriate auxiliary fields [9]. The selective choice of local coordinate systems qualifies M- and L-integrals to be used to distinguish between the material forces of two defects [10].

In this paper, new global approaches of path-independent integrals are presented and applied to the calculation of crack paths at interacting cracks systems. The resulting paths are compared to standard methods being in very good agreement.

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Crack initiation in elliptically notched plates

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Notches or flaws are ever-present in engineering structures. Fracture analyses are required to assess the criticality of these stress concentrators under the expected loading conditions. In this work, the general case of an elliptical notch in a plate under uniaxial tensile loading is considered. Crack initiation at the stress concentration is studied by using a Finite Fracture Mechanics approach [4]. It is shown that the analysis comprises the limit cases of a vertical and a horizontal crack. In these cases the present analysis reverts to a strength of material approach or to Linear Elastic Fracture Mechanics. When intermediate aspect ratios of the ellipse are considered, a smooth transition of a stress-governed failure to an energy-governed failure is observed. Size effects that show a dependence on the size of the hole, which does not change the actual stress fields, are covered by this approach.

To implement the Finite Fracture Mechanics approach, the stress field in the notched plate and the stress intensity factor of cracks emanating from the notch are required. Whereas the former can be obtained in closed-form [2, 1], the latter cannot be calculated analytically. In literature, approximate solutions, generally expressed as

$$K_I = F\sigma\sqrt{\pi a} \tag{1}$$

can be found [5, 3]. Detailed numerical analyses were performed to study the accuracy of the available form functions F and it was found that they are only valid for very short cracks and limited aspect ratios. A more general form function F has been developed that yields good approximate solutions for a very wide range of aspect ratios and crack lengths. With explicit expression of the stress field and the stress intensity factor, the Finite Fracture Mechanics criterion can be evaluated very advantageously.

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Phase-Field Modeling of Fracture in Anisotropic Media

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In the recent years phase-field modeling of fracture has become of high importance to describe complex crack patterns in all kinds of solid materials. Many of the models assume an isotropic material behavior, which of course is not a meaningful assumption for e.g. biological tissues such as arterial walls. The incorporation of anisotropy in phase-field models is also motivated by the emersion of sawtooth crack patterns. Since the phase-field approach introduces an additional (smeared) phase describing the evolution of the crack, this method is well suited to be extended for anisotropic materials without thinking about an adaption of the discretization techniques.

Anisotropy can be incorporated in several ways, like by an extension of the surface energy, i.e. by making the energy release rate orientation dependent, as considered in Li et al. [1]. Our ansatz is based on a pure geometrical approach, namely on an anisotropic formulation of the crack surface itself by using the theory of tensor invariants. This gives through the minimization of the crack surface a physically reasonable meaning, see Miehe et al. [2]. The proposed formulation also opens a good starting point for more complex phase-field models such as coupling with other physical fields, e.g. fracture behavior in porous media, or the incorporation of higher order terms.

Some representative numerical examples show the applicability of our approach by using a robust algorithm based on operator splits, see Miehe et al. [3] for more details on this.

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A non-isothermal phase-field model for damage in two-phase materials

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The understanding of stability and lifetime of multi-phase materials such as, for instance, alloys requires knowledge about the interplay of the different mechanical properties of each phase under external load or thermally induced internal stresses. A widely used model for spinodal decomposotion that accounts for elastic deformation due to lattice mismatch of the phases is the Cahn-Larché system [1]. As a consequence of these internal stresses microcracks form. A phase-field model that introduces a macroscopic damage variable is used, which yields a subdifferential evolution equation [2]. The resulting rate-dependent damage evolution [3] obeys a Griffith-like criterium when a crack is formed. During heating/cooling processes or through energy dissipation by crack growth [4] large temperature gradients can occur in the material. In order to account for this effect, we extend the model to the non-isothermal case by considering entropy balance. We investigate the presented model using two-dimensional Finite Element numerical simulations. In particular, the interaction of the path with the phase boundaries is examined. Since multiple spatiotemporal scales are involved, the application of adaptive discretization is advantageous.

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A novel treatment of crack boundary conditions in phase field models of fracture

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Computation of complex crack patterns is a challenging task in fracture mechanics and the phase field approach thereto shows excellent capabilities. A scalar phase field variable s degrades the elastic energy density in a regularized representation of cracks in isotropic materials [1]. Different methods are used so far to model the property of a crack to weaken a material only in tension and shear. One approach is the decomposition of strain in negative spherical, positive spherical and deviatoric parts, in which only the energy density of the latter two is degraded by the phase field [2]. Another method is based on the spectral decomposition of strain [3] and the split into positive and negative principal strains. However, both models avoid to take an explicit representation of the crack direction into account so that particular situations lead to stress states which violate the crack boundary conditions.

This contribution presents a new approach which takes the direction of the crack into account. For an exact representation of cracks, the stress component normal to the crack surface should only be non-zero in case of compression and shear stresses acting on a frictionless crack surface should vanish. In order to satisfy these conditions stresses and strains are represented with respect to a coordinate system oriented with the crack. The crack normal direction is computed from the gradient of the phase field

$$\boldsymbol{n} = \frac{\nabla s}{|\nabla s|},$$

which is already available from the description of the smeared surface energy. The degrading function of the phase field acts only on some parts of the elastic energy density

$$\psi^{\mathrm{el}} = \tilde{\psi}(\tilde{\boldsymbol{\varepsilon}}, s) + \hat{\psi}(\hat{\boldsymbol{\varepsilon}}).$$

Only the terms using $\tilde{\boldsymbol{\varepsilon}}$ are degraded by the phase field. The degradation of shear stress requires special attention in case there is no unique gradient. The derivation of the stress tensor leads to an elasticity tensor displaying transverse isotropy with symmetry about the crack normal. The five parameters for transversely isotropic material are replaced by the two elastic constants of the undamaged isotropic material and parameters including the phase field variable s.

The new model is able to treat crack opening and closure in a correct manner. In addition no extra numerical effort has to be done to compute stresses and strains. The performance of the proposed model is demonstrated by means of numerical simulations, including representative tests for stationary cracks as well as simulations for dynamically propagating cracks.

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Phase-Field Modeling of Hydraulic Fracture

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Hydraulically driven fracture has gained more and more research activity in the last few years, especially due to the growing interest of the petroleum industry. Hydraulic fracturing, commonly known as fracking, is a technique to increase hydrocarbon production from the subsurface system of reservoirs. Key challenge for a powerful simulation of this scenario is an effective modeling and numerical implementation of the behavior of the solid skeleton and the fluid phase, the mechanical coupling between the two phases as well as the incorporation of the fracture process.

For this purpose we propose a new compact model structure for the Biot-type fluid transport in porous media at finite strains based on only two constitutive functions, that is the free energy function $\hat{\psi}$ and a dissipation potential $\hat{\phi}$. This formulation is then coupled to a phase-field approach for fracture which characterizes an intuitive and descriptive regularization of a crack surface that converges for vanishing length-scale parameter to a sharp crack. In contrast to formulations with a sharp-crack discontinuities, the proposed regularized approach has the main advantage of a straight-forward modeling of complex crack patterns including branching. In this work, we extend these concepts to the coupled problem of fluid transport in porous media at fracture.

We apply a rigorous geometric approach to the diffusive crack surface, governed by the crack phase field. The irreversibility of the evolution of the crack phase field is modeled through a critical stress based growth function. A modular concept is outlined for linking of the diffusive crack phase field to the poroelastic response of the bulk material. This includes a transition from an isotropic Darcy-type fluid flow in the unbroken bulk material to an anisotropic Poiseuille-type fluid flow within a crack. The modular structure is exploited in the numerical implementation of the proposed formulation by constructing a robust finite element method, based on an algorithmic decoupling of updates for the crack phase field and the state variables of the poroelastic bulk response. We demonstrate the model capabilities and performance by means of representative numerical examples of complex fracturing scenarios in two and three dimensions.

3D ductile crack propagation with the XFEM

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In industrial forming processes metals are subjected to large plastic deformations. Related to the plastic deformation pores will start to grow, merge and finally lead to micro cracks within the material. Macroscopically this is recognized as a loss of stiffness of the structural behavior. This effect is usually modeled in the context of damage mechanics, where in the last fifteen years several theories were established to circumvent the mesh dependent localization. Most of them are based on the introduction of a new degree of freedom for the thermodynamic driving force of the damage. This new degree of freedom is then computed by solving an additional scalar balance equation of *Helmholtz* type. Nevertheless, if damage evolves and is used to predict macro cracks the global stiffness matrix becomes ill conditioned. Furthermore cracks are just represented in a smeared way and their dimension is related to the mesh size and some artificial internal length parameter. To overcome this drawback the damage is transfered to discrete cracks if it exceeds a material depended threshold value. The representation of fracture is modeled with the eXtended Finite Element Method (*XFEM*). This numerical tool enables a nearly mesh independent crack representation. The strong discontinuities are modeled by level set functions and the spatial discretization of the displacement field becomes enriched. The enrichment functions are chosen in a way to enable discontinuities in the displacement and even singularities or nearly singularities in derived quantities. Within the *XFEM* only a local neighborhood of the crack becomes enriched and the overall number of unknowns increases only moderately.
Since classical crack propagation criteria loose their validity in a finite deformation context and inelastic material behavior one has to think about new ones. The argumentation for the chosen damage based criterion is that once the crack is initiated it will propagate if the pores around the crack front will continue to coalesce. So the mechanism for crack initiation and propagation is the same.
The problem of evolving the front is treated with a purely geometric approach, which preserves the property of perpendicular level sets. After the propagation no remeshing is needed, but the enrichment scheme has to be updated in the vicinity of the crack front. There is also a need for adapting the history variables in these elements. This becomes necessary because the integration points in the new crack front elements are replaced in a way to consider the discontinuous character of the displacements within these elements.
The load displacement path of cracking structures is highly challenging. That's the reason why neither an entirely force driven nor a displacement driven simulation is able to follow the path and an arc length approach is used.

Keywords: XFEM,crack propagation, non local damage

Numerical methods for crack loading analyses in quasicrystals

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The quasicrystals (QC), short for quasiperiodic crystals, have long-range orientational order but without translational symmetry in particular direction. With such special atom arrangement the quasicrystal is a new material class besides crystals and non-crystals (amorphous solids). Crystallographically, quasicrystals can be categorised into some subclasses (1D, 2D or 3D) which depends on how many quasiperiodic directions there are, e.g. one dimensional quasicrystals have two periodic atom arrangement directions and one quasiperiodic direction [1]. Since the first discovery of quasicrystals in a man made Al-Mn alloy about thirty years ago, people made great effort to research this kind of outstanding material. The investigation of physical properties of quasicrystals, such as the fracture behavior, electronic properties, hardness etc., is essential extending their fields of application. Today, QC are known to exhibit e.g. a very good wear resistance, low friction coefficients and to possess a very low porosity.

This work focuses on macroscopic cracks in 1D quasicrystalline plates, where the material is governed by constitutive equations under the consideration of quasicrystal linear elasticity theory. Analytically, the explicit solutions for the coupled phonon-phason fields of quasicrystals are derived applying a generalized Stroh formalism [2]. The numerical model and simulation for arbitrary cracks are established in a FE environment, where the constitutive behavior of coupled phonon and phason fields is included in special user elements. Different numerical techniques like a displacement interpretation method or modified crack closure integral are applied in a generalized form to calculate stress intensity factors and energy release rates. The validity of conventional fracture criteria and the influence of coupled fields on the mechanical and fracture behavior of quasicrystals are finally investigated.

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Application of a cohesive zone element for prediction of damages in laminated structures

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In the last decades applications of the repetition tools in production and manufacturing processes have widely increased. These tools have to be coated with the materials which have satisfactory lifetime and degradation resistance. Therefore, the main goal of this study is to investigate antiadhesive and wear resistant coatings made of ceramics, plastics and metals produced by High Power Puls Magnetron Sputtering (HPPMS) technique [1]. A cohesive zone element technique (CZ) is applied to realize the interactions of the coatings and the substrate surfaces (see [2]). This goes along with the investigations of the delamination and failure behavior of the involved surfaces. To see the applicability of the model, several structural simulations are peformed. The developed CZ element model is capable of modeling the separation, the contact and also the irreversible reloading conditions in both normal and tangential directions [3]. The model is further developed to be applicable for different structures including different bonding behaviors, with a higher stability. The talk concludes with a detailed discussion of the numerical results of different material and interface properties.

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Macroscopic damage modeling for silicon nitride

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Modeling the damage of brittle materials is of great importance considering a variety of structural components. Prominent examples are high strength engineering ceramics. The present work is concerned with silicon nitride, a material with increasing importance in industrial applications. Using EBSD data allows to map the microstructure of polycrystallines and provides a tool to examine the grains, grain boundaries and their respective orientation. Based on EBSD data, micromechanical simulations of silicon nitride was performed using a micromechanical thermoelastic model by Wippler (2012). The micromechanical model includes damage for the different constituents of silicon nitride and the respective interface. As a result, the effective material properties of silicon nitride were determined. In the sense of a hierarchical model structure the effective properties were applied to macromechanical, phenomenological damage models for monotonous and cyclic loading. It is, thereby, essential to capture the general anisotropic nature of damage. For this purpose, an anisotropic smeared crack model has been adopted which was originally formulated by Govindjee et al. (1995). This model is extended to include crack-closure based on a geometrical approximation proposed by Ortiz (1985). The monotonous damage model, hereby, uses a damage evolution equation proposed by Govindjee et al. (1995). The cyclic damage model is based on an effective crack evolution for slow crack growth in silicon nitride (Lube et al., 2007) which was reinterpreted in terms of damage. The effect of damage on the effective material properties is shown by making use of the directional dependent Young's modulus. Furthermore the application of the cyclic damage model to a four point bending test of silicon nitride as well as the application of the monotonous model to a threed thrust bearing are discussed.

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A regularization approach for damage models based on a displacement gradient

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Common material models that take into account softening effects due to damage have the problem of ill-posed boundary value problems if no regularization is applied. This condition leads to a non-unique solution for the resulting algebraic system and a strong mesh dependence of the numerical results. A possible solution approach to prevent this problem is to apply regularization techniques that take into account the non-local behavior of the damage [1]. For this purpose a field function is often used to couple the local damage parameter to a non-local level, in which differences between the local and non-local parameter as well as the gradient of the non-local parameter can be penalized.

In contrast, we present a novel approach to regularization in which no field function is needed [2]. For that purpose, the regularization directly affects the displacements. The idea is that with an increasing value of the damage the element's volume will increase as well, which is a result of the softening due to the occurring damage. The increasing volume can then be measured by the divergence of the displacements, on which the gradient is now applied. The lack of any field function and the direct application of the gradient to the divergence of the displacements entails several advantages: the numerical effort is considerably reduced and the convergence behavior is improved as well. Naturally, the numerical results are mesh independent due to the regularization.

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Investigation of an elastoplastic material model coupled to nonlocal damage in an implicit-gradient framework

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Since the pioneering works by Kachanov [1] and Rabotnov [2] in the 50's and 60's of the past century and especially during the last three decades, continuum damage mechanics (CDM) has continuously proven to be a suitable framework for describing degradation processes in materials from a macroscopic viewpoint. By introducing additional internal variables into the formulation in order to capture essential microscopic material defects in an averaged sense, CDM becomes especially attractive for e. g. finite element simulations of large-scale problems involving damage and elastoplasticity. Although different approaches exist on how exactly damage is incorporated into a CDM model, most formulations have in common that the material exhibits strain-softening at a certain stage of the loading history (see e. g. [3]).

Unfortunately, the application of conventional (i. e. 'local') softening models in numerical procedures as e. g. the finite element method may lead to serious mesh-dependent results if the damage becomes sufficiently high and the strains localize into a narrow band (see e. g. [4], [5]). This problem is related to the conceptual simplicity of the material model itself which possesses no length scale entering its constitutive description. Therefore, several solution strategies were proposed in the past to restore the well-posedness of the formulation and ensure mesh-objective results. One specific class of models abandones a classical assumption made in continuum mechanics and exploits the principle of 'nonlocal action' (see e. g. [6]). Physical arguments supporting the hypothesis of the nonlocal character of damage were given e. g. by Bažant [7].

Among the latter type of formulations, the so-called implicit gradient-enhanced damage models have gained attention in more recent years (see e. g. [8], [9]). The main ingredient of these models is an additional partial differential equation for a selected nonlocal quantity which has to be solved in addition to the normal balance equations and which becomes a further unknown of the problem. The damage variable itself is then considered to be functionally dependent on this quantity and becomes therefore 'nonlocal' by its own nature. It can be shown that this type of model is equivalent to a specific nonlocal integral-type formulation [10], but its mathematical description remains strictly local which makes its implementation comparably easy from an algorithmic point of view.

The goal of the current study is to examine the behavior and capabilities of an elastoplastic material model which is coupled to damage in an implicit gradient-enhanced setting. Its suitability to describe the crack development phase in case of ductile materials shall be carefully investigated. Various benchmark problems are considered and corresponding finite element computations are carried out in order to estimate the model's capabilities to produce mesh-independent simulation results.

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A three-dimensional progressive damage model for fiber reinforced composites with an implicit-explicit integration scheme

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This contribution proposes a fully three dimensional continuum damage model to describe the interlaminar and intralaminar failure mechanisms of transversely isotropic elastic-brittle materials under static loading. The constitutive model is derived from an energy function with independent damage variables for fiber, matrix and shear failure, i.e. five different failure modes are considered: fiber tension, fiber compression, matrix tension, matrix compression and shear.
The evolution law is based on energy dissipation within the damage process and the increase of damage is controlled by equivalent displacements for each damage mode. The damage model is implemented in a finite element program (FEAP), taking into account the critical energy release rate to weaken the effect of mesh dependend outcome.
It is a well-known fact that the lack of numerical robustness in nonlinear solid mechanics (such as continuum damage mechanics) using standard iterative Newton-Raphson schemes leads to numerical convergence inconveniences and thus in some cases no useable results can be obtained. An implicit-explicit integration scheme, first proposed by Oliver [3] in isotropic damage models, is used to increase the stability and robustness of numerical simulations and to decrease the computational cost of material failure analyses.

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Effects of a circular defect size and position on the damage progression in hybrid laminates under compressive loading

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The composite structures made from glass fiber reinforced with polymer matrices have resistance to compression considerably lower when holes (defects) are present. The hardness of the polymer matrix, the properties of the interface matrix/fiber and fiber stability are the main factors that affect the resistance to compression of notched laminates. The importance of these factors results in complex damage modes which involve the matrix cracking, delamination, local buckling and shear failure.The importance of the damage tolerance analysis of composite materials was shown by many authors [1]. An approach was presented by Wang [2] related to the delamination problem of the composite laminates edges. Davis and Jones [3] have examined the effect of manufacturing defects on the fracture of composite structures. An experimental and numerical study on the residual stresses in the composite laminates subjected to an impact loading was done by the authors [4]. The progressive evolution of the damage in the laminates under compressive loading containing a circular hole has been studied experimentally [5], it was observed that the rupture was initiated at the instability localized at the surface of the hole edge.

In this paper, the effects of a circular defect size and position on the post-damage response of the FMLs laminates under compressive loading has been studied. Compression tests are performed on composite plates comprising a hole having different diameters (8 mm, 12 mm and 18 mm) and located at different positions (25%, 50% and 75%) in the loading direction. A numerical analysis using the finite element method FEM has permits the identification of the initiation zones and the description of the damage evolution in these laminates. The damage has been Modelled by the combined use of the Cohesive Zone Model [6] (CZM) and the Linde mechanical model [7]. The results of the numerical simulation are in good agreement with the experimental results. The FEM has well predicted the critical damage zones. The failure modes are instantaneous in the plates having a defect on the side, this is explained by the high instability of the hybrid laminate (local buckling) once past to the deformation of the second order and the rupture occurs suddenly. However, for the plates having a defect in the center, a lateral deformation of the first order (global buckling) is observed and the rupture is gradual because the aluminum layers tend to stop the crack growth and a stresses redistribution occurs across the plate.

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3D modeling of crack percolation due to alkali-silica reaction in concrete RVEs

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The alkali-silica reaction (ASR) is a reaction in concrete, which occurs between the alkalis in the cement paste and reactive silica within the aggregates. This type of concrete deterioration has been observed in structures all over the world, notably in mass concrete. Since the reaction is slow, ASR is often detected only after many service years. During the reaction, part of the aggregate material is transformed into silica gel that starts swelling as it adsorbs water. The gel expansion is constrained by the surrounding aggregate or paste material and consequently internal stresses in the structure arise. These stresses cause a macroscopic expansion of the structure, cracking and eventually spalling. From a mechanical point of view this internal degradation can be characterized by a loss of strength and elastic modulus. Studying the evolution and the effects of ASR in massive concrete structures such as dams is of particular importance as they are designed for a long service life and repairs are expensive.

The reaction rate of ASR, *i.e.* the rate at which the silica gel forms, depends on various physical and chemical parameters such as the aggregate type, temperature and relative humidity. However, it has been shown in [1] that the macroscopic expansion of unrestrained concrete and the damage are uniquely related to the amount of gel present in the structure. This observation makes it possible to model the mechanical effects of ASR by using the amount of gel present in structure as an input parameter, without having to take into account the speed at which the gel has formed, if creep is neglected. A 2D model based on this conclusion, in which the cement matrix, the aggregates and the swelling gel are modeled as separate phases, was developed by Dunant in [2]. The author used an isotropic continuum damage approach to model the formation and propagation of cracks in ASR samples. The results obtained with this model show the same qualitative trends observed in experiments of ASR samples under uniaxial load [3]. The simulations furthermore revealed that microcracks are aligned with the loading direction and that external loads accelerate the damage evolution.

In this work Dunant's model is extended to 3D in order to simulate full 3D concrete samples. This allows also to make more accurate predictions for the onset of crack percolation. Massively parallel finite-element simulations are carried out to study the mechanical consequences of ASR in 3D representative volume elements (RVEs). For the first set of simulations an isotropic damage formulation, like in the original 2D model, is used. However, since the loading conditions in the case of ASR are complex, the damage evolution in real ASR samples is anisotropic. In order to capture this effect in the simulations the scalar damage variable has to be replaced by a higher order damage tensor [4]. Consequently, a second set of simulations is run using an anisotropic damage formulation. The results for both sets are compared in order to analyze how the initial damage zone around the gel sites changes with the damage formulation.

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Numerical simulation of the fracturing processes in masonry arches

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Masonry arch structures, and, more generally, vaulted structures, are traditionally assessed using a well-established approach, such as linear elasticity or limit analysis, whereby system behavior at the intermediate stage –which occurs when the material's tensile strength has been exceeded but the collapse mechanism has not yet formed –is disregarded. A more accurate interpretation requires a thorough analysis that can take into account the intermediate cracking stage and uses a constitutive law providing a closer approximation to the actual behavior of the material [1]. In this paper, an evolutionary fracturing process analysis for the stability assessment of masonry arches is presented. This method makes it possible to capture the damaging process that takes place when the conditions evaluated by means of linear elastic analysis no longer apply and before the conditions assessed through limit analysis set in. Furthermore, the way the thrust line is affected by the opening of cracks and the redistribution of internal stresses can be checked numerically [2]. The results obtained with the described approach are compared with a numerical simulation performed with the finite element code Diana (TNO, The Netherlands) adopting discrete cracking with nonlinear cohesive laws. Finally, the case study of the arch of the Mosca Bridge over the Dora River in Turin, Italy, is described.

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A damage-plasticity model to simulate the mode-II fatigue behavior of interfaces between thin adherends

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Interfacial strength deterioration due to fatigue is still a major open issue relevant for a broad range of applications [1], such as delamination of composite materials [2] or more generally fatigue life prediction in bonded joints [3]. In particular, fatigue may induce the accumulation of plastic dislocations and damage at the interface level which, in turn, may lead to the nucleation of micro-cracks. Their gradual coalescence may result in a macro-crack, whose propagation is responsible for interface failure [2] even for applied stresses smaller than the maximum attained in monotonic quasi-static conditions [4].

Numerical methods based on the definition of suitable constitutive cohesive interface laws are one of the most versatile and used approach to simulate the behavior of bonded joints. However, some models employ a large number of parameters leading to a complicate calibration and to a very demanding implementation (e.g. [2]). Moreover, these parameters sometimes do not have a straightforward physical meaning (e.g. [1]). On the other hand, simple models based on simplifying assumptions may not fulfill thermodynamic laws (e.g. [5]).

The present work proposes a novel thermodynamically consistent model for the behavior of interfaces under shear (i.e. mode-II) fatigue loading conditions. The interface behavior is defined coupling damage and associated isotropic-kinematic plasticity. Friction and interlocking at the interface level are neglected. The fulfillment of the second law of thermodynamics, in the form of the Clausius-Planck inequality, is ensured by the definition of a dissipative potential following [6]. This potential rules also the coupling between damage and plasticity. The admissible states domain is formulated restricting the tangential interface stress to non-negative values, which makes the model suitable for interfaces with thin adherends. Linear softening is assumed so as to reproduce, under monotonic conditions, a bilinear mode-II interface law. Two different damage variables govern respectively the loss of strength and of stiffness of the interface. The proposed model introduces only four parameters, i.e. three defining the monotonic mode-II interface law, and one ruling the fatigue behavior. This limited number of parameters and their clear physical meaning facilitate experimental calibration. Model predictions are compared with experimental results on fiber reinforced polymer sheets externally bonded to concrete since this case meets the basic assumptions of the model. Different loading histories are accounted for, and an excellent agreement is obtained.

While more analyses are needed to confirm the obtained results, the approach presented appears to be a very promising tool to study the fatigue life of bonded joints. A simple calibration of the fatigue parameter using a relative small number of experimental tests would enable the extension of the prediction of the fatigue behavior of a bonded joint to conditions different from the tested ones.

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Predicting delamination in multilayered CFRP laminates

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In many technical applications, carbon fiber reinforced plastics (CFRP) are gaining importance due to their very high Young's modulus and low density. These are particularly leveraged in thin shell-like structures of lightweight constructions. The composites examined in this paper consist of multiple layers, each of which is composed of carbon fibers embedded in an epoxy matrix material. Two different kinds of layers are considered: (1) unidirectional CFRP with one family of fibers, and (2) woven CFRP with two families of fibers. In order to describe realistically the material behavior of such composites, a slightly modified version of the meso-mechanically motivated material model proposed by Reese [1] is used, which has been shown to be well suitable for such applications [2].

Structural collapse in fiber composite structures can possibly be caused by different damage modes: matrix transverse cracking, fiber fracture, or delamination. From these, the delamination is particularly important, because it drastically reduces the bending stiffness of the structure and promotes local buckling in case of compressive loads. Including delamination into the computation of composite structures requires the definition of an appropriate criterion for its onset as well as the prediction of its growth after an initial crack has evolved. In the present work, the delamination onset allows the consideration of different modes as well as mixed mode situations. Once the delamination onset criterion is met, the propagation of the delamination is predicted by a damage formulation which acts like a cohesive zone model.

The use of a fully three-dimensional material model strongly suggests using solid elements. On the other hand, due to the application in thin shell-like structures, shell elements should usually be preferred. Therefore, a solid-shell element is applied here, which has been presented by Schwarze and Reese [2, 3] combining the advantages of both solid elements and shell elements at the same time. This solid-shell formulation utilizes a reduced integration scheme within the shell plane using one integration point, whereas a full integration is used in thickness direction. Thus, an arbitrary number of integration points can be chosen over the shell thickness. Thereby, different fiber orientations of the layers can be taken into account easily, since the material parameters can be defined for each integration point separately.

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Fatigue mechanisms of cord–rubber composites

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This article presents a multiscale continuum damage mechanics approach to characterize the fatigue mechanisms of cord-rubber composites. The performance of cord–rubber composite is highly influenced by the properties of interface between them which plays a significant role in stress transfer between cord and rubber matrix. An airspring bellow which consists of layers of rubber and reinforcing cords is considered for this work. The phenomenological material model of the rubber is formulated for the purpose of analyzing the rate dependent behavior under cyclic loading. The rate dependency and hysteretic behavior are characterized by using the concept of internal variables [1]. The implementation of the constitutive formulation for rubber material is done in Abaqus via UMAT.

A representative volume element (RVE) of a cord-rubber composite is employed to analyze microscale damage mechanisms i.e. matrix cracking and interfacial debonding. At macroscale, homogenized damage model is applied to predict the stress-strain response. Within the frame of finite element cohesive zone modeling, interfacial debonding is studied. Furthermore, the relationship between two failure mechanisms of fiber-matrix interfacial debonding and matrix cracking are also discussed [2].

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Determination of the fracture properties from size effect in ceramic-fibers-reinforced PMMA

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This talk deals with numerical analyses of size effects in composite materials. The considered composite material is a PolyMethylMethAcrylate (PMMA) matrix reinforced by ceramic fibers. While the bulk's response of the fibre and that of the matrix are modeled by means of standard finite hyperelastoplasticity, material failure in the form of fibre cracking and debonding between the fibre and the surrounding matrix is captured by the strong discontinuity approach. Since the mechanical model parameters characterizing debonding failure are not known, they are computed through an inverse analysis. This inverse analysis confirms a pronounced size effects of the failure modes.

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On Anisotropic and Nonlocal Damage Models

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The talk investigates the modeling of anisotropic damage based on second and forth order damage tensors. Different approaches like the strain or the energy equivalence principle are compared concerning their similarities and differences. The central difference of these individual approaches are different definitions of effective stress tensors. Advantages and open issues of the individual formulations are discussed from a thermodynamical perspective. In addition, the question is raised to what extend the thermodynamical point of view should influence the model formulation. Also addressed are related restrictions on damage models like thermodynamical consistency and the requirement of monotonically increasing damage.
Possible approaches to couple damage with plasticity models and to regularize mesh-dependency are investigated. The findings are illustrated by means of simple simulation results.

Model Reduction and Clustering Techniques for Crash Simulations

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Nowadays, crash simulation is a core area of vehicle development. Recently, regulations and standards also allow the application of numerical simulation (or Virtual Testing) results instead of hardware tests. In the future, hardware tests are expected to be stint in favor of virtual tests conducted to evaluate the safety of a vehicle. Commercial Finite Element (FE) solvers are used to simulate recurring crash scenarios. In addition, with the promotion of active safety techniques, the so-called pre-crash phase is taken into account, which is over 20 times longer than the actual crash phase. Bearing in mind that crash simulation is one of the most (calculating) time-consuming task in car design, this leads to tremendous simulation times. In order to allow for parallelization on computer clusters, explicit instead of implicit methods are used to solve the occurring differential equations in crash simulation. This already reduces the response time but not the overall computational cost. Therefore, the deployment of model reduction for speed-up, data reduction and energy saving in the computing centers is a logical consequence.

But unfortunately, crash simulations possess all three basic sources of nonlinearities in mechanics: Large deformation, nonlinear material behavior and multiple contact scenarios. Using linear reduction with a nonlinear model would result in an unfeasible error. Therefore, nonlinear techniques like POD-DEIM from [1] are investigated for practicability in crash simulations. It uses the POD-Galerkin projection, which finds $N \ll n$ orthogonal vectors $\mathbf{v}_i$ that approximate a given subspace $\text{span}\{\mathbf{y}_1, \ldots, \mathbf{y}_s\} \subseteq \mathbb{R}^n$ of s snapshots (samples of trajectories) best in the sense that

$$\sum_{i=1}^{s} \|\mathbf{y}_i - \sum_{j=1}^{N} \langle \mathbf{y}_i, \mathbf{v}_j \rangle \mathbf{v}_j\|_2^2 \tag{1}$$

is minimal. These $\mathbf{v}_i$ can be calculated as the columns of the left projection matrix $\mathbf{V}$ of the singular value decomposition of $(\mathbf{y}_1, \ldots, \mathbf{y}_s)$. Substitution of $\mathbf{y}(t)$ with $\mathbf{V}\tilde{\mathbf{y}}(t)$, $\tilde{\mathbf{y}}(t) \in \mathbb{R}^N$, in a system of nonlinear ODEs

$$\frac{d}{dt}\mathbf{y}(t) = \mathbf{L}\mathbf{y}(t) + \mathbf{F}(\mathbf{y}(t)) \tag{2}$$

separated into a linear part $\mathbf{L} \in \mathbb{R}^{n \times n}$ and a nonlinear function $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$ leads to

$$\frac{d}{dt}\tilde{\mathbf{y}}(t) = \mathbf{V}^T\mathbf{L}\mathbf{V}\tilde{\mathbf{y}}(t) + \mathbf{V}^T\mathbf{F}(\mathbf{V}\tilde{\mathbf{y}}(t)). \tag{3}$$

Since the nonlinear part $\mathbf{f}(t) := \mathbf{F}(\mathbf{V}\tilde{\mathbf{y}}(t))$ of the system (3) still scales with the high-dimensional model after applying POD, it is essential to apply e.g. the Discrete Empirical Interpolation Method (DEIM) to overcome this problem. The POD-DEIM algorithm from [1] selects only a few, say m, indices from the vector-valued function $\mathbf{f}$ with a projection matrix $\mathbf{P}$ in such a way that a certain error bound can be established. The projection is then given as

$$\hat{\mathbf{f}}(t) = \mathbf{U}(\mathbf{P}^T\mathbf{U})^{-1}\mathbf{P}^T\mathbf{f}(t) \tag{4}$$

with $\mathbf{U}$ the matrix containing the projection basis constructed from snapshots of the full-order function. The reduced system

$$\frac{d}{dt}\tilde{\mathbf{y}}(t) = \mathbf{V}^T\mathbf{L}\mathbf{V}\tilde{\mathbf{y}}(t) + \mathbf{V}^T\mathbf{U}(\mathbf{P}^T\mathbf{U})^{-1}\mathbf{F}(\mathbf{P}^T\mathbf{V}\tilde{\mathbf{y}}(t)) \tag{5}$$

has e.g. the computational complexity $\mathcal{O}(N^2 + mN)$ if the evaluation of $\mathbf{F}$ and its derivative at p components scales linear in p.

It is unnecessary and computationally too expensive to reduce the complete model with nonlinear techniques. Therefore, in a first step, parts with linear and nonlinear characteristics are identified. The identification could either be performed by an experienced user or by the application of methods from machine learning. First the

crash will be simulated with small parameter variations multiple times and the result data is exported from the LS-DYNA binary database to Matlab with a self-written converter. Now clustering methods like the k-means algorithm, PCA or spectral clustering can be used to identify parts of the model that behave similar, c.f. [2] and [3]. This will allow the application of nonlinear methods like the aforementioned POD-DEIM to parts with large deformations and linear methods based on Krylov subspaces and Gramian matrices, which are implemented in the software MOREMBS [4] developed at our institute, to the remaining parts. The size of the local reduced basis can also be decreased since similar behavior often implies a smaller solution space.

Whereas the well-known k-means algorithm has some restrictions, e.g. each cluster is always located in a convex subset of the underlying space, spectral clustering is more flexible. The algorithm summarized in [5] works as follows: First similarities between all pairs of nodes with the position data at each time step and simulation run for different parameters need to be defined, c.f. [3] for details. These will be converted to a l-nearest neighbor graph with l chosen around the logarithm of the number of nodes. As a next step, the first k eigenvectors (with k the number of desired clusters) of the normalized graph Laplacian are calculated and collected in a matrix, whose rows are clustered with the k-means algorithm. This procedure returns clusters of nodes with similar deformation and is justified by heuristic comparisons with graph cutting problems and spectral perturbation theory.

A high-detailed FE model of a 2001 Ford Taurus by the National Crash Analysis Center serves as a validation model. However, the usage of the complete Finite Element model for developing new methods is too complex. Therefore, only a high deformed part like a longitudinal chassis beam is used in place of the complete model to test different clustering algorithms.

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Mechanical properties of cold sprayed Ti- and Ni-based composites coatings

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Cold spray is a safe and innovative coating technology characterized by high efficiency and low cost if compared to the traditional technologies. Very fine particles are accelerated into a De Laval nozzle and sprayed on a substrate. The severe plastic deformation, experienced by the sprayed particles, occurs at temperature well below the melting point leading to the unique mechanical properties experienced by such kinds of coatings. It has potentials to be applied in aerospace, automobile, chemical industry. Cold spray appears very promising in the production of nanostructured composites with enhanced microstructural and mechanical properties. In the present paper the microstructural and mechanical behavior of Ti- and Ni-based nanocomposites coatings produced via cold spray are presented. The results are analyzed as a function of spray velocity and they are compared to those belonging to the unreinforced base materials.

Finite Element Method for the Vibration of Cracked Beams with Varying Cross Section

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This contribution presents ideas, the vibration analysis of the non-uniform cracked beams has been investigated. A computer program using the finite element method has been written to find the dynamical characteristics (natural frequencies and mode shapes) of the beam. The cracked section in the beam has been modeled by a massless spring whose flexibility depens on the local flexibility induced by the crack. The stiffness of spring has been derived from the linear elastic fracture mechanics theory as the inverse of the compliance matrix calculated the using stress intensity factors and strain energy release rate expression. Some examples has been given to explain the proposed method and investigate the effects of the depth and location of cracks on the natural frequencies and mode shapes. The results of current study and those in the literature are compared and good agreements have been found. Consequently it is showed that proposed method is reliable and simple.

Growth of a substrate material damage as a result of waves localization

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Some types of a laminated safety glass consists of an interlayer surrounded by two thin films (see in [1]). An optimal level of adhesion is then required for the safety glass to absorb enough of the impact energy to prevent projectile penetration and delamination. Such considerations have motivated us to study the behavior of such structures undergoing shock like loading and introduce the mathematical model of the structure behavior.The structure under consideration is modeled as a string, an elastic foundation of which is assumed to be imperfect, and which coefficient depends on a damage function of a substrate.The imperfection of an elastic foundation is modeled by a damage function for which the evolution equation is introduced.The governing equation is as follows:

$$\gamma u_{xx} - K(n)u - \rho_0 u_{tt} = Q(x,t,\epsilon), \quad t > 0 \tag{1}$$

where u is a displacement, Q is an external force.

$$Q(x,t,\epsilon) = \delta_\epsilon(x - x_0)\sum_{j=0}^{M}\delta(t - j\Delta t), \quad x \in (-\infty,+\infty), \quad t > 0. \tag{2}$$

Here Δt is a time step for the strikes and δ_ϵ is a smoothed δ, function. Here $K[n]$ is a complicated functional of u that involves a damage function n. The quantity K depends on n via the following relations

$$K(n) = \mu(n)G(n), \quad \mu(n) = \frac{k_0}{k_0 + G(n)} \tag{3}$$

where $k_0 > 0$ is a constant, $G(n) = G_0(1-n), \quad 0 \le n \le 1$; $G(n) = 0, \quad n > 1$, Here G_0 is a constant. The time evolution of the damage function $n(x,t)$ is defined by the differential equation

$$\frac{\partial n}{\partial t} = \beta H(\mu(n)|u| - \Delta)(1-n), \tag{4}$$

where $\beta, \Delta > 0$ are positive constants, and H is the Heaviside step function. We assume that Q is a fast decreasing in $|x|$ smooth function and we set the boundary and initial conditions $u(x,t) \to 0 \quad (|x| \to \infty); u(x,0) = 0, \quad u_t(x,0) = 0, \quad x \in (-\infty,+\infty); n(x,0) = n_0(x), \quad x \in (-\infty,+\infty)$ where n_0 also is a fast decreasing in $|x|$ smooth function.If $x \in [-h,h]$, we set the Dirichlet boundary condition $u(h,t) = u(-h,t) = 0$. If β is small, then $dn/dt << 1$ and the coefficient K is a function of the slow time $\tau = \beta t$. Therefore,we can use a quasi-stationary approximation by the two-time scales perturbation method. We assume that $\beta << 1$ and $K(n)$ is a function of the slow time $\tau = \beta t$. For each fixed τ, there occurs a Schrodinger operator associated with this problem,

$$\mathcal{H}\Psi = \frac{d^2\Psi}{dx^2} - \Phi(x,\tau)\Psi = E\Psi, \quad \Phi = \gamma^{-1}(K(n(x,\tau)) - \bar{k}_0). \tag{5}$$

The eigenfunctions Ψ may be localized ones, when $\Psi = \Psi_j(x,\tau),\ E = \gamma^{-1}(\bar{k}_0 - \rho_0^{-1}\omega_j^2)$, where $j = 1,...,N(\tau)$, and may belong to the continuous spectrum. The analysis of the obtained asymptotic solution shows that for all times there always exists a localized eigenfunction and, thus, $N(\tau) \ge 1$. Also a possibility of resonances between the time periodic external strike load and localized modes has been found.The obtained solution consist of two terms: an oscillating term and a wave term. Main effects are as follows: the damage zone in substrate can cause the non-stationary wave localization in it, and there are possible transitions between different types of the front destruction dynamics. Three scenario of $n(x,t)$ behavior are possible: a monotone growth of n;a piecewise like constant growth of n (when n increases during some intervals and n is constant between these growth intervals); no growth of n.

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